the surface of the earth. This divergence between intuitive concepts based on plane geometry and the realities of spherical geometry (especially for long-range distances) leads to two possible definitions of a curved-surface "straight" line. On a flat surface, a straight line has two properties: (1) it is the shortest distance between two points, and (2) it maintains the same direction (angle) all along its path; psychologically, we take these two properties for granted. On a curved surface like the earth, however, it turns out that we can choose only one of these two properties to define a "straight" line. As a result, two definitions of a "straight" path on a curved surface emerge: the great circle (the line of shortest distance) and the rhumb line (the line of constant direction).

Great Circle

The shortest distance between two points on a sphere is along a great circle, or orthodrome, defined as a "circle on a sphere produced by any plane which passes through the center of the sphere" (Raisz 1962, 292) and through the two points in question. If one point is due north or south of the other—that is, if both points lie along the same meridian (those lines of longitude that converge at the north and south poles)—then the great circle connecting the two points is the meridian itself along which both points lie. More typical is the case of an oblique great circle, a great circle connecting two points (not on the equator) with different longitudes. One answer to the question, then, of what is the direction of another point elsewhere on the globe is to say that it is the initial compass direction—known as the azimuth—of the great-circle path, starting at the initial location. That is, in what direction would we start traveling if we were to trace the shortest path (the great circle) to the destination point.

This particular definition of a "straight" path on the surface of a globe emphasizes the notion of distance. For if the "true" distance between two points, even on a curved surface, is to mean anything, this argument—popular among geographers and mathematicians—goes, it must mean the shortest distance (i.e., along the great circle) between those two points (Reid 1963; Kramer 1970; Robinson et al. 1995). This definition is also the consensus among Muslims in choosing a direction in which to face Mecca (King 1986) and among Bahá'ís for facing Acre (Brown and Bromberg 2000).

To compute the initial azimuth (angle), \( \theta \), between the line extending due north from point 1 and the great-circle route connecting points 1 and 2 on a sphere, the following equation is used:

\[
\theta = \arctan \left( \frac{\sin \Delta \phi}{\cos \phi_1 \tan \phi_2 - \sin \phi_1 \cos \Delta \phi} \right),
\]

In this equation \( \Delta \phi \) is the absolute value of the difference in longitude between the two points (minimum of 0° and maximum of 180°), and \( \phi \) is the latitude of point 1 (\( \phi_1 \) of point 2); note that, in this equation, \( \phi \) should be a negative number for latitudes south of the equator. The solution provided by this equation was first determined in Damascus by the 14th-century astronomer al-Khalili, who developed a qibla table for each degree of longitude and of latitude in the Muslim world (King 1986).

Choosing the initial direction of a great-circle route, however, does have some drawbacks. For one, the navigator thinks of an oblique great-circle course as a line of inconstant direction. Though it is indeed the shortest, most direct route between two points on the earth's surface, you must be ever changing your compass direction with respect to those converging meridians if you would stick to the oblique great-circle route" (Greenhood 1964, 130). In other words, the initial compass direction of a great-circle route will be incorrect as soon as the journey begins, because an oblique great circle's direction (with respect to the north-south meridians) is different for every point along the route (see Fig. 1). This lack of consistency between the initial direction of the great circle versus subsequent compass headings along it seems to violate part of what it means for a path to be "straight": it must maintain the same direction (angle) all along the line. A related difficulty arises when we examine the special case of two points on the earth that are due east or west of each other. In this special case, a person at the more western location who believes that a "straight" path, first and foremost, should have a constant direction, would face due east along the same line of latitude shared by the city he or she is facing, even though that path is not the shortest. This reasoning is probably closer to the views of the 3rd-century Jewish rabbis who said to face eastward when one is west of Jerusalem (Tosefta Brachot 3:16).

Rhumb Line

In contrast to a great circle, a rhumb line, or loxodrome, is a "line which crosses the successive meridians at a constant angle" (Raisz 1962, 296). In other words, a path connecting two points on the earth along a rhumb line—though it will likely not be the shortest path—will maintain the same constant compass direction all the way along the path. Thus, a second definition of a "straight" line on a